Invest or Fall Behind: Maintaining Quality in Hotelling Markets

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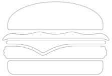
Maintaining Quality under Shocks




- "Dain's Place" vs. "Dimsum Asian Bistro":
  - Consumers have different tastes over food options.

- Quality shocks hit unpredictably:
  - Fryer or oven breakdowns, chef leaves.
  - Restoring and maintaining quality is usually costly.

- Common in other industries as well:
  - A TV show loses its leading star.
  - An app suffers from a recently spotted bug.

Research Questions

Horizontal Differentiation	Vertical Differentiation
Heterogeneous Consumers	Product Quality Differences

- How do firms dynamically invest to maintain product quality, facing heterogeneous consumers?
  - Does consumer heterogeneity intensify or dampen firms' competition in quality?
  - How does this interaction depend on the cost of quality investment?
  - Do firms invest in quality efficiently, over-invest, or under-invest?

First Look at the Model

- Two firms engage in dynamic quality competition (vertical, endogenous) …
- …in Hotelling markets (horizontal, exogenous).
- High-quality products face negative shocks from nature.
- Firms incur a cost to maintain high quality – "product upgrade".

- In each period, each firm decides whether to upgrade, faces nature's potential shocks, and chooses a price.

Preview of the Results

Upgrading Frequencies

- The upgrading frequency is non-monotonic in upgrading costs.
  - Maskin and Tirole (1987, 1988a, b); Pakes and McGuire (1994); Ericson and Pakes (1995); Rosenkranz (1995); Doraszelski and Markovich (2007); Doraszelski and Satterthwaite (2010); Besanko et al. (2010); Board and Meyer-ter Vehn (2013); Abbring et al. (2018).
  - Aghion et al. (2005); Gowrisankaran and Rysman (2012); Eizenberg (2014).
  - Often relying on additional modeling features, such as learning by doing or exit scrap value.
- Two upgrading modes, upgrading deterrence and open competition, emerge from modeling horizontal differentiation by Hotelling markets.

Preview of the Results

Welfare Implications

- Lower or higher upgrading cost: Firms under-upgrade.
  Intermediate upgrading costs: Firms over-upgrade.
  - Mankiw and Whinston (1986); Jones and Williams (2000); Ahuja and Novelli (2017).
  - Bloom et al. (2013); Goettler and Gordon (2011).
- A single model features both under- and over-investment, tied to the investment cost. Under-investment happens at two disjoint cost ranges.

Preview of the Results

Interactions of Differentiation

- Lower upgrading cost: as horizontal differentiation $\Uparrow$, less vertical differentiation. Higher upgrading cost: as horizontal differentiation $\Uparrow$, more vertical differentiation.
  - Shaked and Sutton (1982); Motta (1993); Degryse (1996); Irmen and Thisse (1998); Vanhaecht and Pauwels (2005); Gabszewicz and Wauthy (2012).
  - Two-period, with backward induction arguments.
- Horizontal differentiation changes investment dynamics:
  - Strengthen the quality competition if the competition is already strong.
  - Weakens the quality competition if the competition is already weak.

Outline of the Talk

The Model

Vertical differentiation only

- Benchmark:
  social planner
- Duopoly competition
- Welfare implications

Interaction of differentiation

- Benchmark:
  social planner
- Duopoly competition
- Welfare implications

Extensions

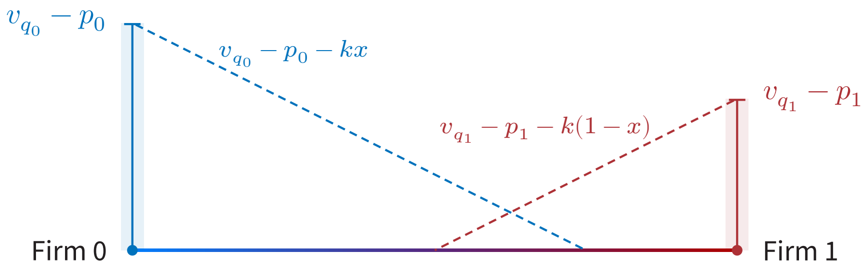
The Model

Firms

- Two long-lived firms, located at two ends of a Hotelling market $[0, 1]$.
  - Firms' locations are fixed at 0 and 1.
- Discrete time, infinite horizon, period length Δ.
  - Δ models how fast firms can take actions.
  - Consider firms can react fast: $\Delta \to 0$.
  - Allowing for cleaner exposition and easier interpretations of the results.
- Each firm produces a product with high or low quality at zero cost:

$$q_i \in \{L, H\}, \quad v_H = 1, \quad v_L \in (0, 1).$$

Consumers



- At each period, there are mass Δ consumers uniformly distributed on $[0, 1]$.
- Horizontal differentiation: Linear transportation cost k. Assume $k \leqslant 1/3$.
- Consumers are short-lived, and each consumer purchases at most 1 product.

Pricing: Without Transportation Cost

$k = 0$

- Reduce to Bertrand competition under quality pair (q_0, q_1).
- $\pi_0(H, H) = \pi_0(L, L) = 0$.
- In the imbalanced state (H, L):
  - *Firm 0*: Charges $p_0 = 1 - v_L$ and occupies the market.
  - *Firm 1*: Charges $p_1 = 0$ and makes no sales.

$$\pi_0(H, L) = (1 - v_L)\Delta, \quad \pi_0(L, H) = 0.$$

- There is only one profitable state: being the quality leader.

Pricing: With Transportation Cost

$k > 0$

- Hotelling competition under quality pair (q_0, q_1).
- In the balanced state (H, H):
  - $p_0 = p_1 = k$.

$$\pi_0(H, H) = \pi_1(H, H) = \frac{k}{2}\Delta.$$

  - Both firms can charge higher prices from their more loyal customers. Details

Pricing: With Transportation Cost

$k > 0$

- Hotelling competition under quality pair (q_0, q_1).
- In the imbalanced state (H, L): (with relatively small k)
  - *Firm 0*: Charges $p_0 = 1 - v_L - k$ and occupies the market.
  - *Firm 1*: Charges $p_1 = 0$ and makes no sales.

$$\pi_0(H, L) = (1 - v_L - k)\Delta, \quad \pi_0(L, H) = 0.$$

  - The market leader has to charge lower prices to attract the opponent's more loyal customers. Details

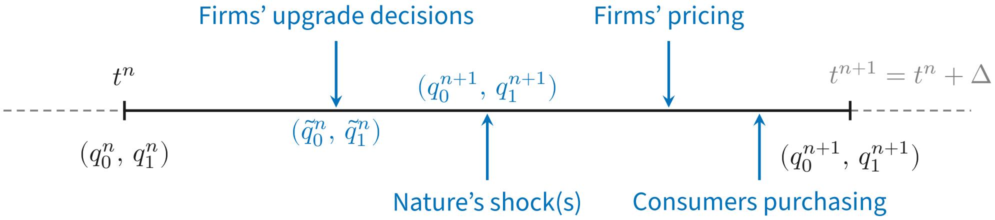
Quality

In each period:

- When $q_i = H$, nature can place a shock leading to quality decay, setting $q_i = L$.
  - Independent shocks between products. Relaxed in an extension.
- When $q_i = L$, firm i can upgrade q_i to H by paying a (lump-sum) cost c.
  - No further shocks from nature if $q_i = L$.

Timing of the Stage Game

Period n



- Results are robust under alternative stage timeline: same results hold if firms can react to nature's shocks in the same period.

Firms' Strategies

- Markov strategies with payoff-relevant state (q_0, q_1).
- Firm i's strategy:
  - Upgrading: when $q_i = L$, making contingent upgrading decisions:

$$\sigma_i : \{(q_i = L, q_j)\} \to [0, 1].$$

  - Pricing (static NE pricing employed):

$$p_i : \{(q_i, q_j)\} \to \mathbb{R}_+.$$

Equilibrium Concept

- Symmetric Markov Perfect Equilibrium (S-MPE). Details
    - In case of multiplicity, we consider the joint-profit maximizing S-MPE.

“Continuous Time” Limit

- Common discount factor $\delta = e^{-r\Delta}$ $\quad\rightarrow\quad$ Discount rate $r > 0$.
- Shock probability $b = 1 - e^{-\beta\Delta}$ $\quad\rightarrow\quad$ Shock rate $\beta > 0$.
- Firms mixed strategies can
  - converge to a rate λ: $\lambda\Delta$ is the approximated mixing probability.
  - converge to a probability that gives immediate state transition.

Vertical Differentiation Only

Social Planner Benchmark

- Suppose $k = 0$.
- A utilitarian social planner maximizes total surplus.
  - Would like the consumer to choose the high quality product if there is one.
  - Set $p_0 = p_1 = 0$ and let the consumers freely choose which product to purchase.
  - Stage social surplus is $\max\{v_{q_0}, v_{q_1}\}$.
- No duplication of high quality as $\Delta \to 0$: At (L, L), should the social planner upgrade to (H, L)?

Social Planner's Optimal Policy

Proposition. The social planner's optimal policy is to keep one product at high quality if

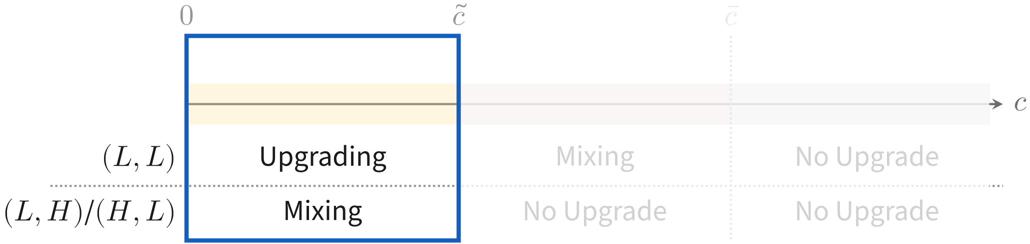
$$\frac{1 - v_L}{r + \beta} \equiv \bar{c} \geqslant c.$$

Otherwise, the social planner's optimal policy is to never upgrade any product.

- The social planner upgrades at (L, L) if the present value of the gain is greater than the upgrading cost. Details

Duopoly Competition

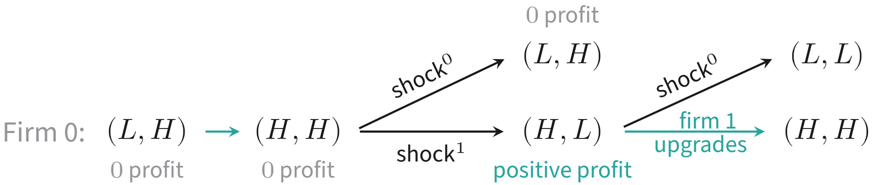
Theorem. There is a unique S-MPE for each $c > 0$ at the limit $\Delta \to 0$.



- When facing homogeneous consumers, firms' upgrading frequency decreases in upgrading cost c.
- Consider the MPE when $0 < c < \tilde{c}$.

MPE at Low Upgrading Costs

Upgrading at (L, L), mixing at $(L, H)/(H, L)$:



- By mixing at (H, L), firm 1 controls firm 0's upgrading incentive by regulating the expected duration of firm 0 at its profitable state (H, L).
- Firm 0 needs to be indifferent and plays the same mixing strategy.
- As $c \Uparrow$, firm 1's mixing $\Downarrow$ to extend the duration at (H, L).

MPE at Low Upgrading Costs

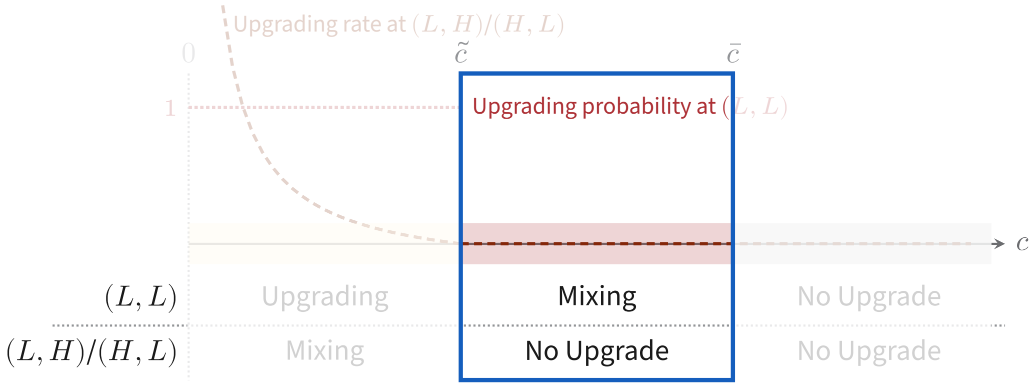
Proposition. If $c \leqslant \tilde{c}$, the following is the limit of a symmetric MPE:

- Firm 0 upgrades at (L, L) for sure.
- Firm 0 upgrade at (L, H) at a rate $f(c)$.

Moreover, f decreases in c.

- The upgrading incentive is provided by the potential future profits only.
- Firms upgrade to keep the opportunity of being the quality leader in the future.

MPE at Low Upgrading Costs



- Consider the MPE when $\tilde{c} < c < \bar{c}$.

MPE at Intermediate Upgrading Costs

- As c increases, firms need even larger upgrading incentives.
- Mixing at (L, L): Mixing realization can be (H, L), so that firm 0's upgrade can have immediate profit.

- As $\Delta \to 0$, the state (L, L) immediately transitions to (H, H), (H, L), or (L, H).

Limiting Behavior of the Mixed Strategy

Let $\hat{g}(\Delta)$ be the upgrading probability at (L, L).

- Does $\hat{g}(\cdot)$ converge to a rate?

$$\lim_{\Delta \to 0} \hat{g}(\Delta) = 0 \quad \text{and} \quad \lim_{\Delta \to 0} \frac{\hat{g}(\Delta)}{\Delta} = \hat{g} > 0.$$

- Suppose firm 1 upgrades with a rate in the limit:
  - At the moment when the state hits (L, L), firm 1 upgrade with 0 probability.
  - Firm 0 should then upgrade for sure, and get to (H, L) for sure.

Limiting Behavior of the Mixed Strategy

$\hat{g}(\cdot)$ must converge to a probability:

$$\lim_{\Delta \to 0} \hat{g}(\Delta) = g \in (0, 1).$$

The possibility of landing at (H, H) counters the first-mover advantage.

- $\mathbb{P}(\text{At least one firm upgrades in a period}) = 1 - (1 - g)^2 > 0$ even if $\Delta \to 0$.
- As $\Delta \to 0$, the state (L, L) immediately transitions to (H, H), (H, L), or (L, H) with probabilities

$$\frac{g^2}{g^2 + 2g(1-g)}, \quad \frac{g(1-g)}{g^2 + 2g(1-g)}, \quad \frac{g(1-g)}{g^2 + 2g(1-g)}.$$

- Larger g: More likely to land at (H, H).

MPE at Intermediate Upgrading Costs

Proposition. If $\tilde{c} < c \leqslant \bar{c}$, the following is the limit of a symmetric MPE:

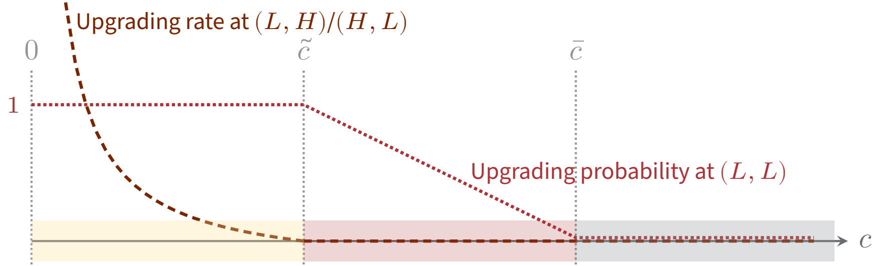
- Firm 0 upgrades at (L, L) with probability $g(c)$.
- Firm 0 does not upgrade at (L, H).

Moreover, g decreases in c.

- More incentives to keep indifference at (L, L) as c increases. More likely to land at (H, L) for smaller g.
- $g(\bar{c}) = 0$: no upgrade at the boundary.

MPE: Vertical Differentiation Only

Theorem. There is a unique S-MPE for each $c > 0$ at the limit $\Delta \to 0$.



(L, L)	Upgrading	Mixing	No Upgrade
$(L, H)/(H, L)$	Mixing	No Upgrade	No Upgrade

Open Competition MPE

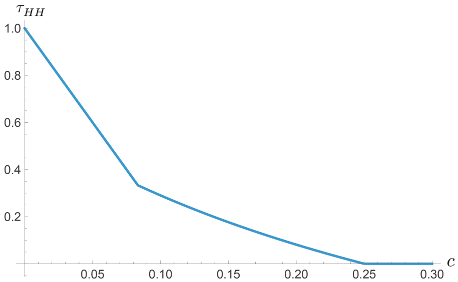
- "Upgrade at (L, L), Mixing at (L, H)" and "Mixing at (L, L), No upgrade at (L, H)":
  - Mixing for correct upgrading incentives at (L, L).
  - Continuity: continuous, decreasing mixing rate / probability and coincide at $\tilde{c}$.
  - Outcome distributions of (H, H), (H, L), and (L, H).

- When there is only vertical differentiation, firms engage in open competition until the cost is too high.

Outcome Distribution

Ergodic outcome distribution of states in MPE:

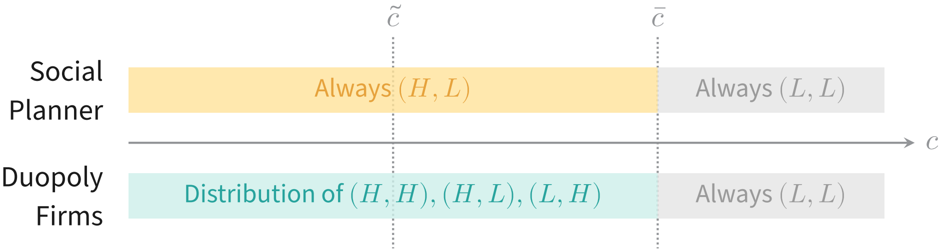
- τ_{HH}: the proportion of time spent at the balanced state (H, H).
- τ_I: the proportion of time spent at the imbalanced states $(H, L)/(L, H)$.
- These measure the extent of vertical differentiation. More



τ_{HH}: Time spent at (H, H)

Over-Upgrading

Corollary. Firms over-upgrade if $c \leqslant \bar{c}$ and never under-upgrade.



Asymmetric MPE Example

Vertical Differentiation Only: Summary

- There is only one competition mode: open competition.
- Firms' upgrading frequency decreases in c.
- Firms over-upgrade compared with the social planner.

Vertical and Horizontal Differentiation

Social Planner Benchmark

- Now consider $k > 0$.
- There are merits to keep both products at high quality.
  - From (H, L) to (H, H), no consumer is worse off, and consumers near location 1 strictly benefit.
- Conjecture: The social planner should upgrade both products if c is sufficiently low.

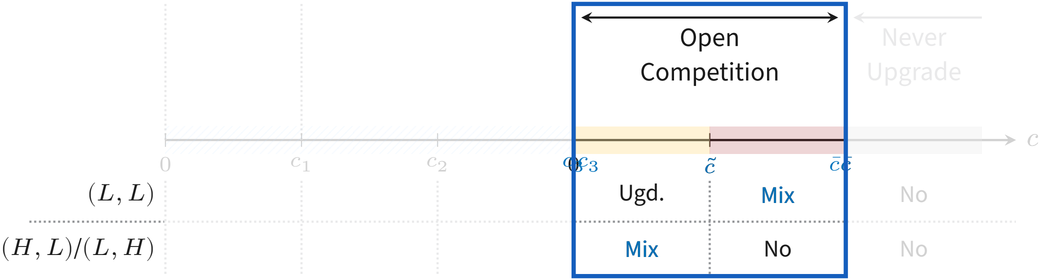
Social Planner’s Optimal Policy

Proposition. There exists $c^A < c^N$ such that social planner’s optimal policy is

Keep (H, H)	c^A	Keep (H, L)	c^N	Keep (L, L)	$\rightarrow c$

- Some policies are never optimal: not consistent when comparing marginal cost and marginal benefit of upgrading. Details

Duopoly Competition



- New competition mode
- New MPE

- Changes in mixing
- Interactions of two dimensions of differentiation

Interaction of Differentiation

Theorem. Suppose $0 < k \leqslant 2/9$ and $0 < v_L \leqslant 1 - 3k$. There exists $\hat{c} \in (c_3, \bar{c})$ such that for a given upgrading cost c,

$$\frac{\partial \tau_{HH}}{\partial k} \geqslant 0 \text{ if } c \in (c_3, \hat{c}], \quad \text{and} \quad \frac{\partial \tau_{HH}}{\partial k} < 0 \text{ if } c \in (\hat{c}, \bar{c}).$$

Interaction of Differentiation

$\partial\tau_{HH}/\partial k > 0$:

$$k \quad \Uparrow \qquad \longrightarrow \qquad \tau_{HH} \quad \Uparrow$$

- Transportation cost.
- Higher horizontal differentiation.

- Time spent at (H, H) in equilibria.
- Less vertical differentiation.

As horizontal differentiation increases, there is less vertical differentiation in the open competition equilibria.

Interaction of Differentiation

$\partial \tau_{HH} / \partial k < 0$:

$$k \quad \Uparrow \qquad \longrightarrow \qquad \tau_{HH} \quad \Downarrow$$

- Transportation cost.
- Higher horizontal differentiation.

- Time spent at (H, H) in equilibria.
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As horizontal differentiation increases, there is more vertical differentiation in the open competition equilibria.

Interaction of Differentiation

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- For $c \in (c_3, \hat{c})$, two dimensions of differentiation are substitutes.
- For $c \in (\hat{c}, \bar{c})$, two dimensions of differentiation are complements.

Interaction of Differentiation

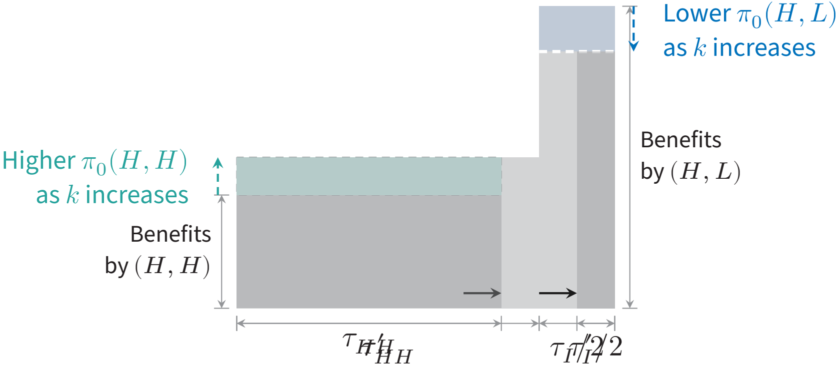
When $k > 0$, *both* (H, L) *and* (H, H) provide upgrading incentives:

- (H, L) still provide stronger incentives: $\pi_0(H, L) > \pi_0(H, H)$.
  - Being the quality leader still grants more profits.
- $\pi_0(H, H)$ increases in k.
  - Firms can charge higher price due to higher market powers.
- $\pi_0(H, L)$ decreases in k.
  - Quality leader chooses lower the price to attract far away consumers and to keep the market dominance.

τ_{HH} decreases in c.

- Quality competition becomes less fierce as c increases.

Substitution at Lower Costs



- Fix $c \in (c_3, \hat{c})$. $\pi_0(H, H)$ and $\pi_0(H, L)$ provide just sufficient upgrading incentives.
- Without changing τ_{HH} and τ_I, overall incentive is larger since $\tau_{HH} > \tau_I$.
- Relocate more time to τ_{HH} since $\pi_0(H, L) > \pi(H, H)$.

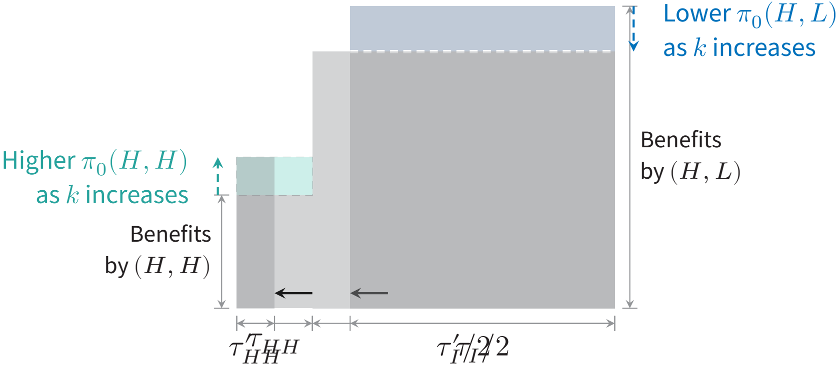
Substitution at Lower Costs

At lower cost levels:

- Firms engage in fierce competition in quality.
- More market power grants more profits and further fuels competition.
- Even more upgrading, leading to less quality differentiation.

Dominant State Enhancing: Firms spend even more time in balanced (H, H) state as horizontal differentiation increases.

Complementarity at Higher Costs



- Fix $c \in (\hat{c}, \bar{c})$. $\pi_0(H, H)$ and $\pi_0(H, L)$ provide just sufficient upgrading incentives.
- Without changing τ_{HH} and τ_I, overall incentive is smaller since $\tau_{HH} < \tau_I$.
- Relocate more time to τ_I since $\pi_0(H, L) > \pi(H, H)$.

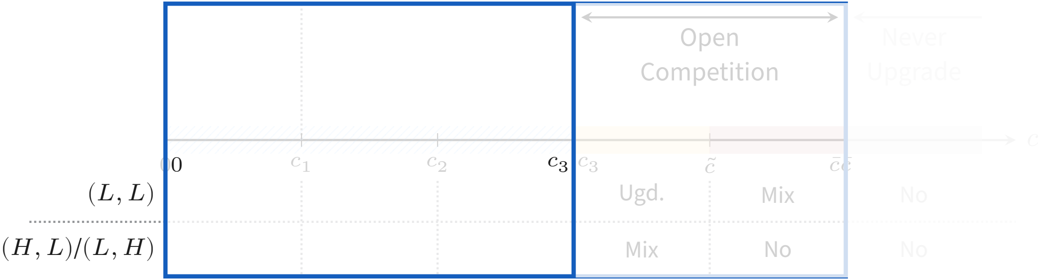
Complementarity at Higher Costs

At higher cost levels:

- Upgrading incentives come from the possibility of being the quality leader.
- Higher market power reduces the gain being a quality leader.
- Lower incentive to compete for the leader position. More likely that one firm upgrades instead of both.

Dominant State Enhancing: Firms spend even more time in imbalanced $(H, L)/(L, H)$ states as horizontal differentiation increases.

Duopoly Competition



- New competition mode
- New MPE

Dominant state enhancing:

- Substitutes at lower cost.
- Complements at higher cost.

Always Upgrade

Proposition. The strategy profile always upgrading when possible is the limit of a symmetric MPE if

$$c \leqslant \frac{\pi_0(H,H) - \pi_0(L,H)}{r+\beta} \equiv c_3.$$

- The upgrading cost is smaller than the present value of gain from the upgrade.
  - And this gain is positive since $k > 0$.
- How about the condition at (L,L)?

State (L, L)

- If firm 1 always upgrade at (L, L):
  - If firm 0 also upgrades:

$$(L, L) \xrightarrow{\text{Upgrading}} (H, H)$$

  - If firm 0 does not upgrade: $(\pi_0(H, H) - \pi_0(L, H))/(r + \beta) \geqslant c$

$$(L, L) \xrightarrow[\text{0 Duration as } \Delta \to 0]{} (L, H) \xrightarrow{\text{Upgrading}} (H, H)$$

- Self-fulfilling: Firms might just upgrade as well at (L, L), as they believe their opponent will upgrade. This suggests possible multiplicity.

Upgrade Deterrence: Lower Cost

- Can firms agree on not upgrading at (L, L)?

Proposition. If $c_1 < c \leqslant c_2$, the following is the limit of a symmetric MPE:

- Firm 0 does not upgrade at (L, L).
- Firm 0 upgrades at (L, H) for sure.

- Competition trigger: A deviation to upgrade triggers a forever quality war.

Upgrade Deterrence: Lower Cost

If Firm 1 Deviates

lower bound c_1 for meaningful deterrence

1 (H, H) forever

Fight a quality war

2 (L, L) forever

Equilibrium state

3 (L, H) then (L, L) forever

One-step deviation: not fight

upper bound c_2 for credible threat

- Horizontal differentiation is necessary: If $k = 0$, 1, 2 and 3 coincide with each other, and no positive range of c supports upgrading deterrence.

Upgrade Deterrence: Higher Cost

Proposition. If $c_2 < c \leqslant c_3$, the following is the limit of a symmetric MPE:

- Firm 0 does not upgrade at (L, L).
- Firm 0 upgrades at (L, H) with a rate $h(c)$.

Moreover, $h(c)$ decreases in c.

- Switch to a (in expectation) finite-length quality war.

Upgrade Deterrence: Higher Cost

- At higher cost levels:
  - Higher self cost: forever quality war is too costly to implement.
  - Higher opponent cost: forever quality war offers more deterrence than necessary.

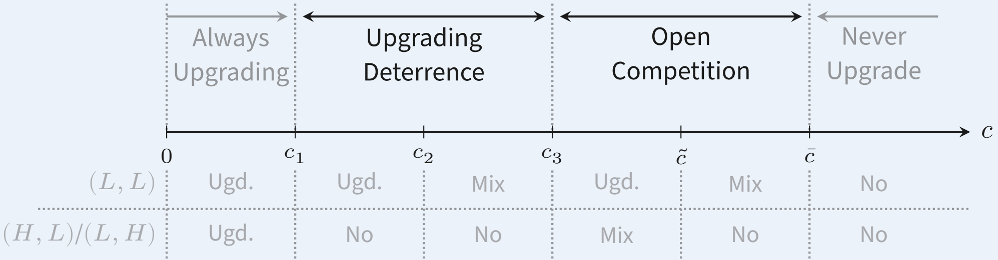
$\Rightarrow$ Switch to quality war with (expected) finite length.

- As c increases, shorter length is required and desired. At c_3, deterrence is too costly to maintain.

- Upgrade deterrence offers higher joint profits compared with always upgrading when possible.

MPE

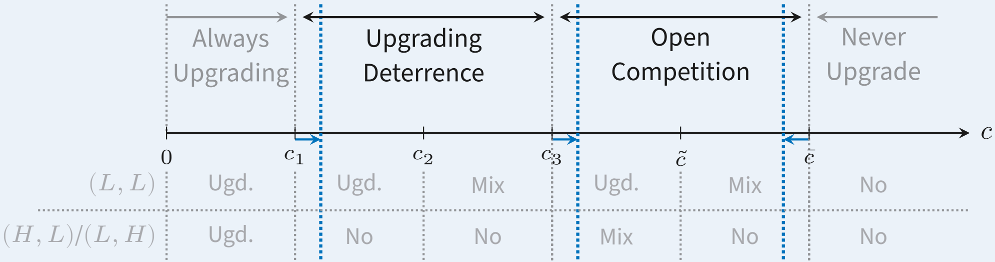
Theorem. The joint-profit maximizing S-MPE in the limit is



- Non-monotonicity of upgrading frequency in upgrading cost.

Higher Horizontal Differentiation

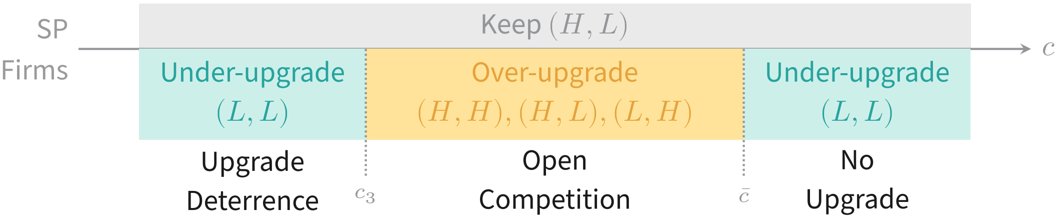
Theorem. The joint-profit maximizing S-MPE in the limit is



- $c_1, c_3 \Uparrow$: Higher profits at (H, H).
- $\bar{c} \Downarrow$: Lower profits at (H, L).

Over- and Under-Upgrading

- While the social planner keeps (H, L):



- Under-upgrade at lower cost level: Upgrade deterrence.
- Under-upgrade at higher cost level: Failure to internalize consumer surplus.

More Others Conclusion

Extensions

Correlated Shocks

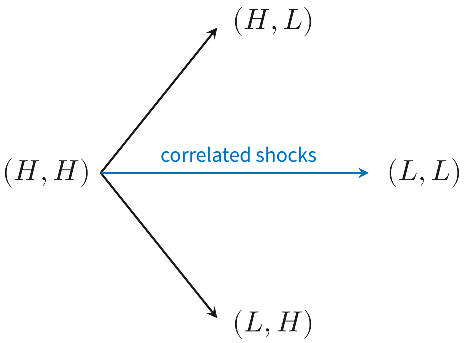
- Shocks between firms can be correlated. Let ρ be the correlation coefficient.

	Shock to Firm 1	No Shock to Firm 1
Shock to Firm 0	$b^2 + \rho b(1-b)$	$b(1-b) - \rho b(1-b)$
No Shock to Firm 0	$b(1-b) - \rho b(1-b)$	$(1-b)^2 + \rho b(1-b)$

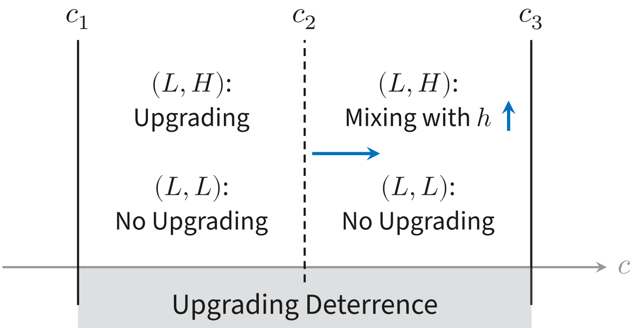
- For this talk: $\rho \in (0, 1]$. Arguments for negative correlations are symmetric.
- ρ remains constant as $\Delta \to 0$.

The Effect of Correlation

- (H, H) cannot transition into (L, L) directly as $\Delta \to 0$.
- (H, H) can transition into (L, L) directly.

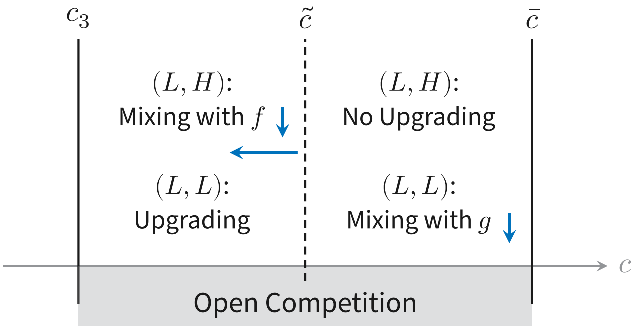


Upgrading Deterrence



- Deterrence is less effective since the punishment phase can be terminated sooner when (H, H) falls to (L, L) directly.
- Finite-length punishment MPE disappears when $\rho = 1$.

Open Competition



- Less upgrading incentives due to skipping (H, L) and (L, H). Upgrading frequencies must be lower to compensate for the loss of incentives.
- First open competition MPE disappears when $\rho = 1$.

Conclusion

Conclusion

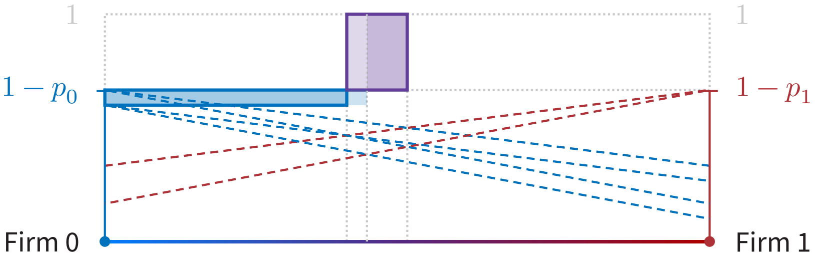
- With horizontal differentiation:
  - Two competition modes: upgrading deterrence and open competition.
  - Non-monotonic upgrading frequency and efficiency.
  - Under-upgrade first, then over-upgrade, then under-upgrade.

- Horizontal differentiation has dominant state enhancing effect:
  - At lower cost levels, horizontal differentiation substitutes vertical differentiation.
  - At higher cost levels, horizontal differentiation complements vertical differentiation.

Thank You!

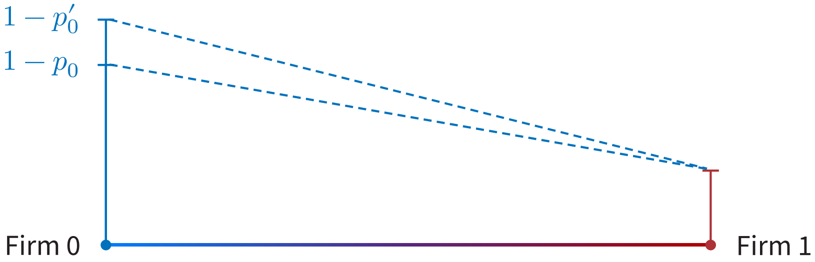
Appendix

Stage Game: With Transportation Cost



- Hotelling competitions under quality pair (q_0, q_1). Consider (H, H) first.
- Balancing higher margin and losing demand when raising price.
- $k \Uparrow$: Less competition, less demand loss from raising price.
- $\pi_0(H, H) = k/2$. Increasing in k. Back

Stage Game: With Transportation Cost



- At (H, L), for v_L not too large, Firm 0 occupies the market.
- $k \Uparrow$: Harder to reach consumers far away, lowering the price.
- $\pi_0(H, L) = (1 - v_L - k)\Delta$. Decreasing in k. Back

Symmetry

- Harsanyi Symmetry-Invariance Criterion.
- Robustness considerations:
  - Fixed costs: Asymmetric equilibria, such as Chicken, cannot survive if there is a (small) fixed cost every period.
  - Evolutionary stability: In each round, a new player is drawn from a large population to take the role.
- Efficiency: Firms are still not efficient in most of the asymmetric equilibria. Examples come later. Back

MPE

- Industry dynamics: Focusing on quality evolution.
- Traditions in theory and empirical literature: Maskin and Tirole's Trilogy (1987, 1988a, 1988b), Ericson and Pakes (1995), Doraszelski and Satterthwaite (2010), Brown and MacKay (2023), Betancourt *et al.* (2024); Bajari, Benkard and Levin (2007); Aguirregabiria, Collard-Wexler and Ryan (2021).
  - Tractability concerns.
- Later: Can implement (seemingly) collusion outcome with MPE already. Back

Social Planner's Problem

No Upgrade at (L, L)

$$W_N(L,L) = \underbrace{v_L \Delta}_{\text{flow}} + \overbrace{e^{-r\Delta}}^{\text{discount}} \underbrace{W_N(L,L)}_{\text{continuation}} \xrightarrow{\Delta \to 0} W_N(L,L) = \frac{v_L}{r}$$

- Stays at (L, L) forever and receives the perpetuity of the flow payoff $q_L = v_L$.

Social Planner's Problem

Upgrade at (L, L)

$$\begin{aligned} W_U(L,L) = \;& -c \quad \text{upgrading cost} \\ \text{no shock} \;& +e^{-\beta\Delta}\left[1\Delta + e^{-r\Delta} W_U(H,L)\right] \\ \text{shock} \;& +\left(1-e^{-\beta\Delta}\right)\left[v_L\Delta + e^{-r\Delta} W_U(L,L)\right] \end{aligned}$$

$$\begin{aligned} W_U(H,L) = \;& e^{-\beta\Delta}\left[1\Delta + e^{-r\Delta} W_U(H,L)\right] \\ & +\left(1-e^{-\beta\Delta}\right)\left[v_L\Delta + e^{-r\Delta} W_U(L,L)\right] \end{aligned}$$

$$\xrightarrow{\Delta \to 0} \quad W_U(L,L) = \frac{1}{r} - \frac{(\beta + r)c}{r}$$

- Stays at (H, L) forever, receives the perpetuity of the flow payoff 1, and pays the costs. Back

Firms' Long-Run Average Joint Profits

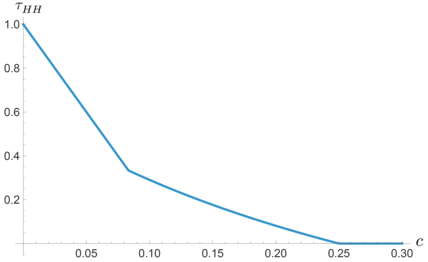
- We can also use τ_B and τ_I to calculate firms' long-run average joint profits.
- Long-run: free of the influence of the initial state.
- Long-run average profit is defined as

$$2\tau_{HH}\pi_i(1,1) + \tau_I\left[\pi_i(1,v_L) + \pi_i(v_L,1)\right] - \mathbb{E}(\text{upgrading cost}).$$

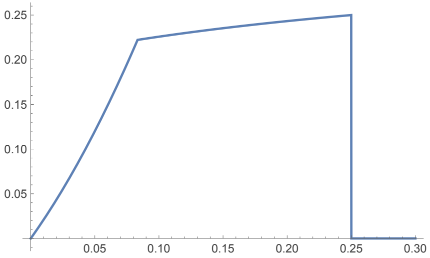
where the expectation is calculated according to the upgrading frequency of each state in the MPE.

Firms' Long-Run Average Joint Profits

Proposition. Firms' long-run average joint profit is 0 if $c = 0$ or $c > \bar{c}$. At $0 < c < \bar{c}$, firm's long-run average joint profit is increasing in c.



Time spent at (H, H)



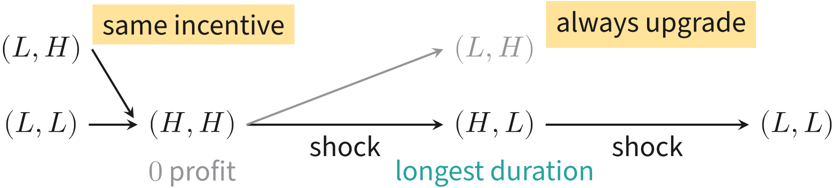
Firms' joint profits

Back

Asymmetric MPE Example

No Chicken (One firm upgrades at (L, L) and no one upgrades elsewhere) MPE if $c < \tilde{c}$:

- In Chicken Firm 1 does not upgrade at (H, L).
- Firm 0 has strict incentive to upgrade at (L, H) and (L, L), even if Firm 1 upgrades at (L, L) for sure.



Asymmetric MPE Example

Proposition. If $c_3 < c < \tilde{c}$, there exists an (asymmetric) MPE with the limit of the following form:

- Firm 0 upgrades at (L, L) for sure and upgrade with a rate at (L, H).
- Firm 1 upgrades with a probability at (L, L) and does not upgrade at (H, L).

- The range of c supporting this MPE coincides with the first open competition MPE.
- When $k = 0$, $c_3 = 0$.

Asymmetric MPE Example

Firm 0 upgrade at (L, L) for sure and upgrade with a rate at (L, H).
Firm 1 upgrade with a probability at (L, L) and does not upgrade at (H, L).

- Firm 0's strategy is the same as in the symmetric MPE: Firm 1 is best responding.
- Firm 1 mixing with probability at (L, L) offers Firm 0 a larger upgrading incentive at (L, L).

- States on path: (H, H), (H, L), and (L, H). Inefficiency due to competition.
- Harder to describe, but minor new insights. Back

Social Planner's Problem

Upgrading both products at (L, L) but no product at (H, L):

- Upgrading both product at (L, L): Marginal Benefit of the Second High $\geqslant c$.
- No upgrade at (H, L): Marginal Benefit of the Second High $\leqslant c$.

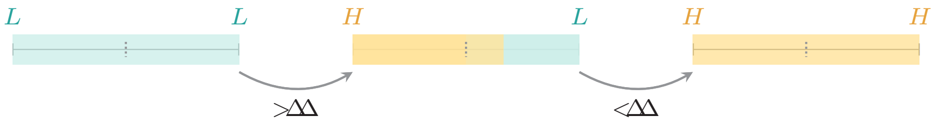
Upgrading one product at (L, L) and one product at (H, L):

- Upgrading one product at (L, L): Marginal Benefit of the Second High $\leqslant c$.
- Upgrading one product at (H, L): Marginal Benefit of the Second High $\geqslant c$.

Social Planner's Problem

No upgrading at (L, L) but upgrading one product at (H, L):

- Suppose the gain from (L, L) to (H, H) is 2Δ.
- If the firms evenly divide the market:
- But consumers re-allocate:



- The gain from the first high-quality product is higher than from the second. Back

Firms' Long-Run Joint Profits

Increasing c has two effects:

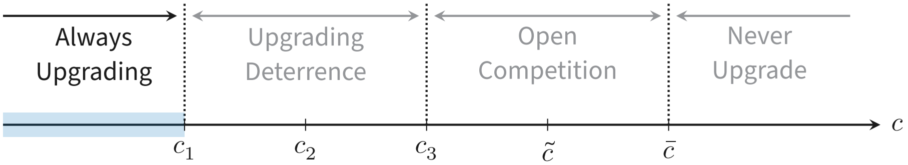
Cost Effect
Each upgrade is more expensive

- $\Downarrow$ joint profits.

Competition Effect
Upgrading is less frequent

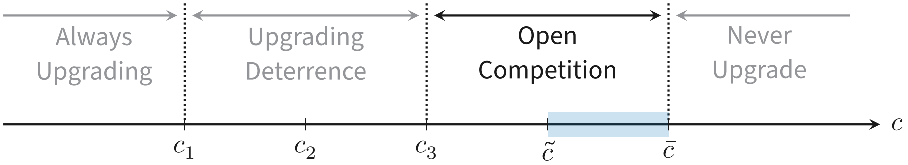
- $\Uparrow$ joint profits.
- Dominating when $k = 0$.
- Influenced by the size of k.

Firms' Long-Run Joint Profits



- Decrease in c initially:
  - For $k > 0$, the existence of market power enables an always-upgrading region.
  - Only direct cost effect presents hence dominates.

Firms’ Long-Run Joint Profits



- Can decrease in c for larger c, if shocks are frequent enough:
  - Firms can maintain this MPE for more frequent shocks due to complementarity.
  - Direct cost effect is stronger when shocks are frequent enough. Back

Efficiency at Other Cost Ranges

- Sufficiently low costs: both the social planner and the firms always upgrade. Efficient.
- First upgrading deterrence: the steady state outcome depends on the initial state.
  - Initial state not (L, L): steady state at (H, H), efficient.
  - Initial state (L, L): steady state at (L, L), under-upgrade.
- Sufficiently high costs: both the social planner and the firms never upgrade. Efficient. Back